Numerical Simulation of Pumping Tests in Multilayer Wells with Non-Darcian Flow in the Wellbore

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Abstract

An important feature of pumping in a multilayer system is the dynamic interaction between the aquifers via non-Darcian vertical flow within the wellbore. An equivalent hydraulic conductivity (EHC) approach is used to include this transient interaction as a special leakage through the confining layer in the flow system. Different flow regimes (laminar and turbulent flows, and the transition range) are involved in a multilayer pumping test where the relation between the hydraulic gradient and flow velocity is complicated. The change of flow regimes is accommodated by varying the content of EHC as a function of the friction factor. For a particular well, the relation between friction factor and Reynolds number is unique and is obtained by interpolating Nikuradse's experimental results. A quasi-three-dimensional Galerkin finite-element method is used to integrate the vertical one-dimensional flow in the wellbore and confining layers and the two-dimensional flow in the aquifers. This approach makes it possible to couple the aquifer and wellbore flows of different flow regimes and to solve them in a single numerical framework based on the same linear relation between gradient and velocity. A carefully designed pumping test in a multilayer aquifer system near Beihai City, Guangxi Autonomous Region, China, is used to demonstrate the performance of the approach.

Introduction

In most geologic formations, aquifers and confining layers are interlayered. In a wellfield, pumping wells commonly penetrate several aquifers to increase well productivity. For research purposes, it may be technically possible to install a pumping well with a screen only in one aquifer in a multiaquifer system and test the different aquifers separately, but this is expensive. The common case where wells penetrate more than one aquifer has to be considered. There are two important issues associated with pumping in a multilayered system. First, there is a dynamic interaction between the aquifers via vertical flow through the wellbore. Second, the vertical flow through the borehole is often non-Darcian, a process not adequately addressed by existing literature.

There have been studies about pumping tests in multiaquifer systems since the 1950s (e.g., Sokol 1963; Papadopoulos 1966; Wikramaratna 1984, 1985; Rathod and Rushton 1991). These studies assumed that all the head loss occurs within the aquifer and that the head along the wellbore is constant. In reality, a significant head loss due to vertical flow in the pumping well can exist (e.g., Jacob 1947; Rorabaugh 1953). For a pumping test conducted in a multilayer aquifer system, the flow states in the wellbore should be represented correctly to estimate the aquifer parameters properly. This is because different flow states in the wellbore lead to differ-

ent head loss, which controls the vertical flow through the wellbore and then the partitioning of the total pumping to individual aquifers. There are hypothetical studies that include the vertical flow in numerical models (e.g., Cooley and Cunningham 1979; Butler et al. 1994; Sudicky et al. 1995), but these studies assume that the flow is either laminar or fully turbulent. Interested readers can see Kohl et al. (1997) and Sudicky et al. (1995) for reviews of these studies. A model is required to simulate not only the laminar and turbulent flow, but also the transitional flow. This transitional behavior restricts the validity of the Darcy or quadratic relation. To our knowledge there is no reported case study of the transitional flow regime in a wellbore under real pumping conditions.

This paper presents an approach for the simulation of multilayer pumping tests with non-Darcian flow through the wellbore. By introducing EHC, the approach uses pipe flow theory to describe the behavior of the vertical flow through the wellbore and couples this behavior with ground water flow through aquifers and leaky confining layers. In this way, the pumping rate is partitioned automatically into the discharges of individual aquifers and the dynamic interaction between aquifers due to transient vertical flow through the wellbore is simulated. The change of flow regimes in the wellbore is accommodated by varying the content of EHC as a function of the friction factor. For a particular well, the relation between friction factor and Reynolds number is obtained by interpolating the Nikuradse's experimental results (Streeter 1961). After incorporating the wellbore flow into vertical leakage across confining layers, the problem is approximated by a quasi-three-dimensional Galerkin finite-element method. The performance of the approach is demonstrated by application to a pumping test conducted in a multilayer aquifer system in Guangxi Autonomous Region, China.

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Received February 1998, accepted November 1998.

Figure 1. A conceptual multilayer aquifer system under pumping.

Discharge Rates and EHC in Multilayer Wells

Discharge Rates in a Multilayer Pumping Well

Figure 1 shows a pumping well taking water from two confined aquifers. The configuration of this flow system is based on the field study to be presented later. The rate of discharge of the well is equal to the sum of the rates of water flow into the well from two aquifers and the rate of decrease in volume of water within the well, giving (Wikramaratna 1984):

$$Q = Q_1 + Q_2 - \pi r_w^2 \frac{\partial H}{\partial t} \quad (t > 0) \quad (1)$$

where r_w is the internal radius of the well; Q is the constant discharge rate of the well; Q_1 and Q_2 are the rates of the two aquifers (note: a positive value represents extraction from a layer, whereas a negative value represents injection into a layer); and $\partial H / \partial t$ is the rate of change in water level in the well.

The last term on the right-hand side of Equation 1 represents the well storage, which may have significant influence on pumping results if the well radius is large and the test period is short. A detailed discussion about this can be found in Jiao and Rushton (1995). The focus of this study is the vertical flow through the wellbore, not storage of the wellbore. The well radius for the case study in this paper is about 0.07 m and pumping lasted 4.5 days. Wellbore storage is therefore ignored, although there is no technical difficulty in including wellbore storage in the model.

When wellbore storage is negligible, Equation 1 becomes

$$Q = Q_1 + Q_2 \quad (2)$$

If Q_1 and Q_2 can be separated and specified, a well of this type can be represented as a group of single-layer wells, each open to one of the layers tapped by the multilayer well, and each having an individual discharge. Different approaches have been used to separate Q into Q_1 and Q_2 . The most common approach is the one suggested by the authors of MODFLOW (McDonald and Harbaugh 1988), which divides the well discharge in proportion to the layer transmissivities:

$$Q_1/Q_2 = T_1/T_2 \quad (3)$$

Intuitively, this approach seems correct because greater transmissivity; generally leads to greater discharge. However, the method lacks a sound theoretical basis and may produce unreasonable results. For example, T_1/T_2 is a constant for a particular system, but it is difficult to believe that Q_1/Q_2 is also a constant. Some researchers (Elbel et al. 1992; Sudicky et al. 1995) present methods that can calculate automatically transient discharge partitioning as a function of aquifer properties. For example, Sudicky et al. (1995) assume that vertical flow in the wellbore is Darcian, and they included it as an integrated part of their model by superimposing conductive one-dimensional line elements representing the well screen onto the three-dimensional matrix elements representing the aquifer. The approach used in this paper is similar to that of Sudicky et al. (1995), but is more general in the sense that it can handle non-Darcian flow by introducing an EHC for the wellbore that varies with velocity.

EHC for Wellbore Flow

The vertical flow through the wellbore is similar to pipe flow. Based on pipe flow theory, the relation between head loss (ΔH) and mean velocity (u) in a circular pipe can be expressed by the Darcy-Weisbach formula (Streeter 1961):

$$\Delta H = f \frac{l}{d} \frac{u^2}{2g} \quad (4)$$

where l is the length of the pipe; d is the internal diameter; u is the average velocity of water in the pipe; f is the friction factor; and g is the acceleration due to gravity.

For laminar flow:

$$f = 64 / R_e \quad (5)$$

where R_e is the Reynolds number:

$$R_e = ud/\nu \quad (6)$$

where ν is the kinematic viscosity of the fluid:

$$\nu = \mu / \rho \quad (7)$$

where μ is the dynamic viscosity and ρ is the density.

Substituting Equations 5 through 7 into Equation 4 gives

$$u = \frac{d^2 \gamma \Delta H}{32 \mu l} = \frac{d^2 \gamma J}{32 \mu} \quad (8)$$

where J is the hydraulic gradient and g is specific weight. Equation 8 indicates a linear relation between ΔH and u .

Notice that for pipe flow, porosity $n = 1$ and Darcy velocity $V = nu = u$, Equation 8 can be written as

$$V = \frac{d^2 \gamma}{32 \mu} J \quad (9)$$

Darcy's law is expressed as

$$V = KJ \quad (10)$$

Figure 2. Nikuradse's experimental results for pipes of uniform and grain roughness (Streeter 1961).

Comparison of Equations 9 and 10 leads to

$$K_1 = \frac{d^2 \gamma}{32 \mu} \quad (11)$$

where K_1 is defined as the hydraulic conductivity for pipe flow under laminar flow conditions. Equation 11 was obtained by Chen (1966) and Bear (1972) independently.

Equation 11 provides an important basis for this study. Taking a multilayer well system as shown in Figure 1, the part of the leaky confining layer that the well penetrates can be regarded as a special geologic material with hydraulic conductivity expressed as Equation 11, and the recharge (Q_2) through the wellbore from the lower aquifer (II) to the upper aquifer (I) can be regarded as special leakage. This can be expressed as

$$Q_2 = K_1 \frac{\Delta H_w}{l_{12}} \pi r_w^2 \quad (12)$$

where ΔH_w is the head difference between aquifer I and aquifer II at the well center, and l_{12} is the distance between the two aquifers. During the time of pumping, vertical flow Q_2 through the wellbore occurs naturally and depends on the transient heads in the aquifers, as does the leakage through the leaky confining layer. The recharge Q_2 , together with the leakage between the aquifers, can be solved numerically. After Q_2 is estimated, Q_1 can be easily obtained as $Q_1 = Q - Q_2$.

The flow regime can be divided into different states, as shown by the Nikurades curves (Streeter 1961) in Figure 2. The linear relation (Equation 8) is valid only for laminar flow when $Re = (Re)_{critical} < 2300$ (corresponding to zone I in Figure 2). When Re is greater than 10^5 , flow becomes fully turbulent (zone V). The friction factor f is independent of Re but is dependent on the relative roughness, which was defined as the size (e) of sands coated on the internal pipe surface divided by the pipe diameter (d) (Streeter 1961). In this case, ΔH is a function of u^2 , as can be seen from Equation 4. In the range where $2300 < Re < 10^5$, the flow behavior is transitional. The transitional range can be further divided into three zones (zones II through IV). The flow behavior in this range is complicated and the influence of Re and the relative roughness on the friction factor cannot be described by simple equations. In ground water hydrology, there are numerical models based on either lam-

inar or fully turbulent flows (zones I or V), but there is no model that can handle flow in the transition range due to the difficulties in expressing the relation between ΔH and u .

When nonlaminar flow (including fully turbulent and transitional flows) is involved, the traditional linear Darcy's law with a constant hydraulic conductivity cannot be used and the simulation of flow may be complicated. However, if hydraulic conductivity is redefined in the following way, the traditional linear form of Darcy's law can be maintained and conventional numerical methods can still be used.

Equation 4 can be rewritten as

$$V = u = \frac{2 g d}{u f} \frac{\Delta H}{1} = \frac{2 g d}{u f} J \quad (13)$$

If the hydraulic conductivity is defined as

$$K_n = \frac{2 g d}{u f} \quad (14)$$

Then Equation 13 can be put into the form of Darcy's law:

$$V = K_n J \quad (15)$$

where K_n may be regarded as the hydraulic conductivity for the wellbore under nonlaminar conditions. In this case, Equation 12 should be modified as

$$Q_2 = K_n \frac{\Delta H_w}{l_{12}} \pi r_w^2 \quad (16)$$

K_n is a kind of equivalent hydraulic conductivity because it varies with velocity and is different from a constant parameter in the traditional sense. The constant hydraulic parameters for laminar pipe flow and ground water flow may be regarded as special cases of the EHC. Therefore, by introducing the EHC (denoted as K_e), Darcy's law can be expressed in a general way:

$$V = K_e J \quad \text{where} \quad \begin{cases} K_e = K & \text{For Darcy flow in aquifer material} \\ K_e = K_1 & \text{For laminar pipe flow in wellbore} \\ K_e = K_n & \text{For nonlaminar pipe flow in wellbore} \end{cases} \quad (17)$$

Using the EHC, all the relations are represented by the same linear form. The change of flow conditions (seepage, laminar, and nonlaminar wellbore flows) is accommodated by varying the EHC, which depends on the friction factors corresponding to the different flow regimes. Because f in Equation 13 or 14 is not written explicitly, these equations can be used for any flow regimes shown in the Nikuradse curves in Figure 2. If the friction factor is known, this approach makes it possible to couple the aquifer and vertical wellbore flows of different flow conditions in a seamless way and to solve them in the same numerical framework. The remaining question is how to obtain f .

For a particular well, the wellbore material and well radius are known. Therefore, the relative roughness can be determined. The relation between the friction factor and the Reynolds number is unique and could be obtained using Nikuradse's experimental method. Nikuradse presented the f and Re relation for pipes with various relative roughness (Figure 2). For a well with relative

Figure 3. Change of friction factor with Reynolds number in observation and pumping wells interpolated from Nikuradse's experimental results in Figure 2.

roughness different from those in Figure 2, the f and Re relation for that well can be interpolated based on the existing Nikuradse curves (note: for this study, the Nikuradse curves reproduced by Streeter [1961] were used). For example, Figure 3 shows the f and Re relation for wells with relative roughness of 0.000328 and 0.000714 interpolated from Figure 2. The friction factor for any given Re or velocity in the two wells can be further interpolated from the Nikuradse curves in Figure 3. After the friction factor is obtained, the EHC can be calculated using Equation 14.

Unlike the real aquifer hydraulic conductivity, the EHC for non-laminar flows varies with time for the wellbore because it is dependent on the vertical velocity. Because Re is dependent on u , and u in turn is dependent on f , which is dependent on Re , an iteration scheme is required to determine the relations among the three parameters. Figure 4 shows a flow chart of the iteration process.

Numerical Representation of Ground Water Flow

The aquifer system under consideration has three aquifers separated by two confining layers (Figure 1). It is assumed that the system can be represented by a quasi-three-dimensional model where flow is horizontal in the aquifers and vertical in the confining layers (Bredehoeft and Pinder 1970). Because the model will be employed for pumping test analysis, drawdown is used in the following equations instead of hydraulic head. Assume that the drawdown in the unconfined aquifer (Figure 1) is negligible and the bottom of aquifer II is impermeable. The horizontal boundary is considered to be sufficiently far from the pumping well that during the test period; drawdown beyond this distance is negligible. The boundary value problem for this system is expressed by the following equations:

For unconfined aquifer:

$$s_0(x, y, t) = 0 \quad (x, y) \in D_0, 0 < t \leq t_e \quad (18)$$

For confining layer 1:

$$\frac{\partial}{\partial z} \left(K'_e \frac{\partial s'}{\partial z} \right) = S'_s \frac{\partial s'}{\partial t} \quad (x, y, z) \in D', 0 < t \leq t_e \quad (19)$$

$$s'(x, y, z, t)|_{t=0} = 0 \quad (x, y, z) \in D' \quad (20)$$

$$s'(x, y, z, t)|_{z=M_2+M''+M_1+M'} = s_0(x, y, t) \quad (x, y) \in D' \cap D_0, 0 < t \leq t_e \quad (21)$$

$$s'(x, y, z, t)|_{z=M_2+M''+M_1} = s_1(x, y, t) \quad (x, y) \in D' \cap D_1, 0 < t \leq t_e \quad (22)$$

For aquifer I:

$$\begin{aligned} \frac{\partial}{\partial x} \left(T_1 \frac{\partial s_1}{\partial x} \right) + \frac{\partial}{\partial y} \left(T_1 \frac{\partial s_1}{\partial y} \right) - K'_e \frac{\partial s'}{\partial z} \Big|_{z=M_2+M''+M'} \\ - K''_e \frac{\partial s''}{\partial z} \Big|_{z=M_2+M''} + \sum_{i=1}^{N_w} \frac{Q_i}{F_i} = S_1 \frac{\partial s_1}{\partial t} \end{aligned} \quad (x, y) \in D_1, 0 < t \leq t_e \quad (23)$$

$$s_1(x, y, t)|_{t=0} = 0 \quad (x, y) \in D_1 \quad (24)$$

$$s_1(x, y, t)|_{(x, y) \in B_{1,1}} = 0 \quad 0 < t \leq t_e \quad (25)$$

$$T_1 \frac{\partial s'_1}{\partial n} \Big|_{(x, y) \in B_{1,2}} = 0 \quad 0 < t \leq t_e \quad (26)$$

For confining layer 2:

$$\frac{\partial}{\partial z} \left(K''_e \frac{\partial s''}{\partial z} \right) = S''_s \frac{\partial s''}{\partial t} \quad (x, y, z) \in D'', 0 < t \leq t_e \quad (27)$$

$$s''(x, y, z, t)|_{t=0} = 0 \quad (x, y, z) \in D'' \quad (28)$$

$$s''(x, y, z, t)|_{z=M_2+M''} = s_1(x, y, t) \quad (x, y) \in D'' \cap D_0, 0 < t \leq t_e \quad (29)$$

$$s''(x, y, z, t)|_{z=M_2} = s_2(x, y, t) \quad (x, y) \in D'' \cap D_2, 0 < t \leq t_e \quad (30)$$

For aquifer II:

$$\frac{\partial}{\partial x} \left(T_2 \frac{\partial s_2}{\partial x} \right) + \frac{\partial}{\partial y} \left(T_2 \frac{\partial s_2}{\partial y} \right) - K''_e \frac{\partial s''}{\partial z} \Big|_{z=M_2} = S_2 \frac{\partial s_2}{\partial t} \quad (x, y) \in D_2, 0 < t \leq t_e \quad (31)$$

$$s_2(x, y, t)|_{t=0} = 0 \quad (x, y) \in D_2 \quad (32)$$

$$s_2(x, y, t)|_{(x, y) \in B_{2,1}} = 0 \quad 0 < t \leq t_e \quad (33)$$

$$T_2 \frac{\partial s'_2}{\partial n} \Big|_{(x, y) \in B_{2,2}} = 0 \quad 0 < t \leq t_e \quad (34)$$

where s , M , S , S_s , and T are drawdown, thickness of the aquifers or the leaky confining layers, storativities of the aquifers, specific storage of the leaky confining layers, and aquifer transmissivity, respectively (subscripts 0, 1, and 2 stand for unconfined, upper, and lower confined aquifers and prime and double primes represent upper and lower leaky layers); K_e is EHC, which varies with the

Figure 4. Flow chart for calculation of friction factor by iteration.

location of the confining unit (inside or outside wellbore) and the flow states (Equation 17). Q_i and F_i are the pumping rate and section area, respectively, of the pumping well i under the assumption that the pump intake is in or above aquifer I (otherwise Q_i should be included in Equation 31 for aquifer II). The rate will be partitioned automatically into the discharges of aquifers I and II based on Equations 2 and 12 (or 16). N_w is the total number of multilayer wells and t_e is the total time period of pumping. $B_{1,1}$ and $B_{2,1}$ are the boundaries of aquifers I and II with zero drawdown; $B_{1,2}$ and $B_{2,2}$ are the flux boundaries of aquifers I and II. D_0 , D_1 , and D_2 are the domains of unconfined, upper, and lower confined aquifers; D' and

Table 1
Description and Thickness of Aquifers
and Confining Layers in the Test Area

Aquifer or Confining Layers	Description	Thickness (m)
Unconfined aquifer	Primarily gravel	2.0 to 4.0
Confining layer 1	Clay, with stringers of sand and gravel	12.2 to 14.0
Aquifer I	Mainly gravel, with lenses or clay or clayed material	13.5 to 17.6
Confining layer 2	Clay, with stringers of sand and gravel	9.4 to 13.8
Aquifer II	Mainly gravelly sand and sand, with lenses of clay or clayed material	23.8 to 26.1
Confining layer 3	Clay	10.0 to 20.0
Aquifer III	Silt	Unknown

D'' are the domains of confining layers 1 and 2; and D is the total domain ($D=D_0+D'+D_1+D''+D_2$).

The quasi-three-dimensional Galerkin finite-element method developed by Fujinawa (1977) is used to solve the flow problem in both the leaky confining layers and the aquifers. Application of Galerkin's method to Equations 18 through 34 leads to a set of algebraic equations. These equations can be expressed in matrix form with matrices such as the conductance and leakance matrices. The leakance matrices include variables such as discharge rates Q_1 and Q_2 . The rates are dependent on ΔH_w and EHC at the wellbore, which is also controlled by hydraulic heads (see Equation 16). Therefore, unlike those used by Fujinawa, the leakance matrices here change with time and can be determined only after the friction factor is estimated and the EHC is calculated. This is achieved by iteration. In the iteration process, the friction factor and the leakance matrices are first estimated based on ΔH_w calculated from the previous time step (see the iteration scheme shown in Figure 4). After that the model is run again to generate new ΔH_w and compare it with the old ΔH_w . Only when the difference is smaller than a certain criterion, the model proceeds to the next time step. Details about implementation of this approach can be found in Chen et al. (1992).

Case Study

A pumping test was conducted in a multilayer coastal aquifer system near Beihai City, Guangxi, P.R. China (see inset in Figure 5). Before the test, the natural change in hydraulic head caused by regional flow and tidal fluctuation was observed in order to correct the drawdown during pumping. The equivalent hydraulic conductivity approach is used to deal with vertical flow through the wellbore of both the pumping and observation wells. The quasi-three-dimensional method was used to simulate ground water flow in the multilayer aquifer system.

Site Background

The stratigraphy of the test site is described in detail elsewhere (Chen et al. 1992). The sediments are Jurassic and Quaternary coastal deposits consisting of interlayered clays, silts, sand, and gravel. Based on their hydrogeologic features, the sediments can be divided into an unconfined aquifer, two confined aquifers, and two leaky confining layers (Figure 5). The extension and the thickness of these layers are based on site investigation and borehole log-

Figure 5. Hydrogeological sections in a multilayer aquifer test area near Beihai City, Guangxi Autonomous Region, China.

Figure 6. Distribution of pumping and observation wells in the test area.

Table 2 Observation Wells in Aquifers and Their Distance to the Middle Point of the Two Pumping Wells		
Observation Well	Aquifer Penetrated	Distance to the Middle Point of Two Pumping Wells (m)
GK1-2	Aquifer I	6.9
GK2-2	Aquifer I	16.1
GK3-1	Aquifer I	37.1
GK1-3	Aquifer II	6.9
GK2-3	Aquifer II	16.1
GK3-3	Aquifer II	37.1
ZK91	Aquifers I and II	11.0
GK3-4	Unconfined aquifer	37.1

ging. Some details about these layers are presented in Table 1.

The main recharge source of the aquifer system is precipitation. The general ground water flow direction is from northeast to southwest. Aquifers interact with each other via leakage. The ground water regime is affected by the tidal effect in the sea. The tidal fluctuation in the confined aquifers is 0.1 to 0.6 m and is progressively smaller toward the inland area. The unconfined aquifer, however, indicates little tidal effect. Before the pumping test, the water levels in the unconfined aquifer and confined aquifers I and II were 5.26 m, 3.87 to 3.93 m, and 3.96 to 3.98 m, respectively.

Figure 7. Observed (circles) and simulated drawdown (lines) during both pumping and recovery periods.

Table 3
Rate of Natural Decline, Total Drawdown, Extrapolated Natural Decline During the Test Period of 4.5 Days, and the Ratio Between Natural Decline and Drawdown at the End of Pumping

Aquifer Penetrated	Observation Well	Rate of Natural Decline (m/d) [A]	Drawdown at the End of Pumping (m) [B]	Extrapolated Total Natural Decline (m) 4.5A	4.5 A/B
Aquifer I	GK1-2	0.010	0.880	0.045	0.051
	GK2-2	0.007	0.760	0.031	0.041
	GK3-1	0.007	0.620	0.004	0.051
Aquifer II	GK1-3	0.012	1.290	0.054	0.042
	GK2-3	0.011	1.200	0.050	0.041
	GK3-3	0.010	0.980	0.045	0.046
Aquifers I and II	ZK91	0.008	1.120	0.036	0.032
Unconfined aquifer	GK3-4	0.000	0.000	—	—

Pumping Test

Two multilayer pumping wells of the same well radius, ZK90-1 and ZK90-2, were installed into aquifers I and II. The intake of the pumps is located in aquifer I. The horizontal distance between the two wells was 1.90 m. There were eight observation wells (Figure 6), only one of which penetrated both aquifers. An attempt was made to install the wells with screen over the entire thickness of the aquifer. Some details of the wells are provided in Table 2. The test was conducted during a dry period between December 11 to 16, 1988. The pumping rate for both wells was 1697 m³/d. During the pumping period, no drawdown was detected in the unconfined aquifer. After pumping, drawdown recovery was monitored until December 22, 1988.

To determine the natural change of hydraulic head with time, observation of water levels started on December 4, six days before the actual pumping test. There was a general trend of water level decline with time of 0.007 to 0.012 m/d (Table 3). The fluctuation of hydraulic head due to tidal effects was less than 0.15 m per cycle.

Because there was a regional decline and a tidal fluctuation, the observed drawdown was corrected before it was used in the model. The component due to tidal fluctuation was regular and was first deducted from the observed drawdown. The natural decline during the test period was extrapolated using the rates in Table 3 and further correction was made by deducting the estimated decline from the observed drawdown. There may be errors in extrapolation, but the error is believed to be insignificant because the extrapolated decline at the end of pumping for most observation wells is less than

Figure 8. Zones of estimated parameters in the test area on the basis of geological grounds (the two pumping wells are located approximately at the center of the circular area).

Figure 9. Temporal change of Reynolds number in pumping wells (a) and observation well (b) during pumping and recovering periods.

Table 4 Diameter, Roughness and Relative Roughness of the Multilayer Pumping and Observation Wells		
	Multilayer Pumping Well	Multilayer Observation Well
Diameter (m)	0.137	0.063
Roughness (mm)	0.045	0.045
Relative roughness	0.000328	0.000714

Table 5a Estimated Hydraulic Conductivity and Storativity for Aquifer Zones 1 to 6				
Aquifer	Geologic Material	Zone No.	K (m/d)	S
I	Gravel	1	67.63	14.4×10^{-5}
	Coarse sand	2	52.60	8.9×10^{-5}
	Gravel and sand	3	37.57	6.3×10^{-5}
		4	30.06	6.3×10^{-5}
II	Clayey gravel and sand	5	51.85	43.2×10^{-5}
	Clayey gravel sand	6	16.01	10.2×10^{-5}

Table 5b Estimated Vertical Hydraulic Conductivity (Zones 1 to 5) and Specific Storage for Confining Layers				
Leaky Confining Layer	Geologic Material	Zone No.	K (m/d)	S'_s and S''_s (l/m)
1	Clay	1	0.0013	3.6×10^{-5}
	Gravelly and sandy clay	2	0.0025	
		3	0.0025	
2	Clay	4	0.0008	
	Gravelly and sandy clay	5	0.0010	

Table 6 Statistics of the Errors in Simulated Drawdowns					
Error (m)	<0.05	0.05–0.10	0.10–0.15	>0.15	Total
No. of readings	212	151	51	6	420
Percentage	51	36	12	1	100

5% of the total drawdown (see the last column of Table 3). The temporal change of drawdown, after correction, is shown in Figure 7.

Nikurades Data on the Wells

There are two multilayer pumping wells and one multilayer observation well. The diameter, roughness, and relative roughness of the three wells are presented in Table 4. As discussed earlier, the Nikurades curves for the observation and pumping wells (with relative roughness of 0.000328 and 0.000714) can be interpolated from the existing Nikurades experimental results and are shown in Figure 3. The friction factor of the wellbore for any Reynolds number can be read from the two curves in Figure 3.

Table 7
Discharges (m³/d) from Aquifers I and II and Transmissivities Near Multilayer Wells

Multilayer Well	Transmissivity (m ² /d)	Discharge at t=0.0007 days	Discharge at t=4.5 days
Pumping well (ZK90-1)	T ₁ =1166	Q ₁ =625	Q ₁ =658
	T ₂ =1088	Q ₂ =1072	Q ₂ =1039
	T ₁ /T ₂ =1.07	Q ₁ /Q ₂ =0.58	Q ₁ /Q ₂ =0.63
Pumping well (ZK90-2)	T ₁ =1166	Q ₁ =630	Q ₁ =663
	T ₂ =1114	Q ₂ =1067	Q ₂ =1034
	T ₁ /T ₂ =1.05	Q ₁ /Q ₂ =0.59	Q ₁ /Q ₂ =0.64
Observation well (ZK91)	T ₁ =949.4	Q ₁ =5	Q ₁ =22
	T ₂ =1081	Q ₂ =-5	Q ₂ =-22
	T ₁ /T ₂ =0.879	Q ₁ /Q ₂ =-1	Q ₁ /Q ₂ =-1

Numerical Simulation

A circular model area is used in the pumping test analysis (Figure 8). The lateral boundary is considered to be 1.6 km from the pumping wells, such that no disturbance due to pumping for 4.5 days would occur beyond it and hence the drawdown at this distance can be set at zero. The unconfined aquifer is used as the upper boundary and the drawdown fixed at zero. The third confining layer, which contains 10 to 20 m of clay, is treated as an impermeable bottom boundary. The initial drawdown is zero everywhere.

The system is discretized into triangular elements. To better depict the rapid change of drawdown near the pumping wells, the elements are progressively finer toward the pumping wells. In each aquifer there are 408 elements and the total elements in the two aquifers is 816. In each layer there are 211 nodes and the total nodes in three layers is 633.

Estimation of Parameter Values

The parameter distributions are divided into zones and shown in Figure 8. This zonation is entirely based on the field investigation (Chen et al. 1992). The values of K and S for aquifer I have three zones, and those for aquifer II have two zones. The values of hydraulic conductivity for leaky confining layer 1 has three zones and that for leaky confining layer 2 has two zones. Due to limited information, both confining layers are assumed to have a uniform specific storage. The total unknown parameters are 18 and will be identified by the numerical model. Preliminary values and ranges for the parameters were initially assigned on the basis of sediment type and known regional hydrogeologic conditions (Chen et al. 1992). Although only approximate at this stage, the assumed values serve to considerably reduce the search effort. Both the trial-and-error method and the Fibonacci optimization method (Chen and Tang 1990) were used to calibrate the model against observed drawdown. The final estimated parameters are shown in Table 5 and the simulated drawdown in Figure 7. The estimated parameter values agree closely with the distribution of the geologic materials (Table 5) and the fit between the observed and simulated drawdowns is satisfactory (Figure 7).

Table 6 shows statistics concerning the error (defined as the absolute difference between the observed and simulated drawdowns) in the simulated drawdown. For the 420 drawdown readings, the majority (51%) of the errors are smaller than 0.05 m and only 1% of the errors are greater than 0.15 m.

Flow and Flow Conditions Through the Wellbore

In the model, both multilayer pumping and observation wells are treated in the same way except that the pumping rate for the observation well is zero. The change in the discharge rates Q_1 and Q_2 from the two confined aquifers in three multilayer wells with time and the estimated transmissivities near the wells are examined and presented in Table 7. As discussed earlier, the ratio between the discharges from two aquifers changes with time and is not equal to the ratio of aquifer transmissivities. The majority of pumped water was from aquifer II. The rate from aquifer II in the observation well was negative, which indicates that in the observation well aquifer I recharged aquifer II.

Figure 9 shows how the Reynolds number in the wells between the two aquifers changes with pumping time. During the pumping period, the Reynolds number for the two pumping wells (Figure 9a) is greater than 10^5 and, therefore, the vertical flow is fully turbulent. In the recovery period, the Reynolds number reduces to zero and flow is fully laminar. The temporal change of Reynolds number in the wellbore of the multilayer observation well (Figure 9b) is relatively smooth. During a large portion of the pumping period, the Reynolds number is around -5000 (note: Reynolds number is positive by definition. The negative value here indicates that water flows from aquifer I to aquifer II) and flow is in the transitional range. Flow is laminar at the beginning of the pumping period and in a large part of the recovering period. Figure 9 shows that the flow conditions in the wellbore can be turbulent, laminar, or transitional. The model can describe all the flow states occurred in the entire pumping and recovering periods in the pumping and observation wells.

Conclusions

This paper presents the theory and practice for analyzing pumping tests in a multilayer ground water flow system using the equivalent hydraulic conductivity approach. Usually there is a head difference between different layers that causes vertical wellbore flow. The vertical movement of water in the multilayer well can be non-Darcian and is described based on pipe flow theory. Wellbore flow is treated as a part of the leakage through the leaky confining layer. By introducing the EHC, the same linear Darcy flow equation is used for different flows while the features of these flows are accommodated by varying the EHC. An important step in simulating the nonlaminar flow through the wellbore is to obtain the relation between the friction factor and Reynolds number for the particular well. This is done by interpolating the Nikurades experimental results based on the relative roughness of the wellbore. Because the EHC for nonlaminar flow is dependent on the vertical velocity and therefore varies with time, an iteration scheme is used to determine the EHC. A carefully designed pumping test in a multilayer aquifer system was used for the case study to demonstrate the applicability of this method. The Reynolds number indicates that the vertical wellbore flow can be in any flow states. For the case study, the flow in the abstraction wells in the pumping period is largely turbulent and that in the observation well is transitional. To the authors' knowledge, this paper presents the first interpretation of the field pumping test involving wellbore flow at various flow status. This approach may be used for other aquifer systems in which different flow conditions coexist, such as fractured media and karst systems with pore, fracture, and even conduit flows.

Acknowledgments

The paper is supported by National Natural Science Foundation of China (NNSFC) (No. 49272144) and Committee on Research and

Conference Grants (CRCG) at the University of Hong Kong. The authors are grateful to Scott Bair, Carl Nuzman, Tiraz Birdie, and an anonymous reviewer whose constructive comments lead to a significant improvement of the manuscript. Thanks also go to John Hall for his review of an earlier draft of this paper.

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